

Gaetano Pagani – Giuliano Andrea Pagani

**CONSIDERATIONS DEVELOPED
ON THE TRANSFORMATIONS
OF TIME SERIES BY LINEAR FILTERS:
THEORETICAL AND PRATICAL ASPECTS**

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Key Words

Stochastic processes; autocorrelation function; linear trend; seasonality; heteroscedasticity; linear filters; linear systems; gain function; differences operators; ARIMA models.

Abstract

The purpose of this work concerns the transformations of non-stationary time series using the known methods of filtering, in order to make stationary time series under investigation. It was therefore used the theory of linear systems to highlight the characteristics of the most used filters in the analysis of time series based on stochastic models of ARIMA type.

In support of the first part mainly theoretical, we want to propose a comparative analysis of some well-known filtering operations on a number of time series of economic interest that have all the components of classical analysis of time series: trend in mean, seasonality and heteroscedasticity.

1. INTRODUCTION

In stochastic analysis of time series, the necessary condition to apply this methodology is the stationarity of the process that would be underlying the phenomenon.

A stochastic process, it should be recalled, is defined as a set of random variables $X(t)$, $t \in T$ where T is the set of time points where the process is defined.

A stochastic process is **strictly stationary** when the joint distribution of probabilities of random variables $X(t_1), \dots, X(t_n)$ is the same of $X(t_1+k), \dots, X(t_n+k)$, $\forall (t_i+k) \in T$ and $\forall n$, where T as already mentioned above, denotes the set of points in time where the process is defined.

This implies that the distribution of $X(t)$ should be the same for all t ; so that the mean of the process is:

$$\mu(t) = \mu = E[X(t)]$$

and the variance is:

$$\sigma^2(t) = \sigma^2 = E[X(t) - \mu]^2$$

which are both constant and independent from value of t .

If we consider two variables, the joint distribution of probability of $X(t_1)$ and $X(t_2)$ depends only on the *delay time* (lag) $t_2 - t_1$. The variance is not enough to specify the second moment of a random variable. In reason of that we have to introduce the **autocovariance function**, namely:

$$\gamma(k) = E[(X(t) - \mu)(X(t+k) - \mu)]$$

It depends exclusively on lag time $t_2 - t_1 = k$.

A stochastic process is **weakly stationary** if its mean is constant and its autocovariance function depends only on lag time k , so that

$$\mu = E[X(t)]$$

and

$$COV[(X(t); X(t+k))] = \gamma(k) = E[(X(t) - \mu)(X(t+k) - \mu)]$$

$$\forall t \in T.$$

This also implies that the variance is constant, then:

$$VAR(X) = \sigma^2 = E[X(t) - \mu]^2$$

In case of weakly stationary processes nothing is assumed about the joint probability distribution; if the joint probability distribution of $X(t_1), \dots, X(t_n)$ is normal multivariate for all t_1, \dots, t_n , the stationarity of second order also implies stationarity in the strict sense because the multivariate normal distribution is completely defined by the first and second moment.

The autocovariance function $\gamma(k)$ is indicative of linear relations between random variables at time lag k . It is an even function so

$$\gamma(k) = \gamma(-k)$$

Since then symmetrical about the y-axis, $\gamma(k)$ can be considered only for $k \geq 0$. Since the auto covariance is expressed in the square unit of measurement of the phenomenon, it could take values not defined; for this reason it is difficult to interpret in practice. For this purpose it is customary to standardize $\gamma(k)$; this function is called **autocorrelation function** $\rho(k)$

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)}$$

Since in a stationary stochastic process

$$\gamma(0) = COV[(X(t) - \mu)(X(t-0) - \mu)] = VAR[X(t)] = \sigma^2$$

we will obtain

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)} = \frac{\gamma(k)}{\sigma^2}$$

This function, as is known, has some properties which, briefly, are listed below:

1. for $k=0$ we have $\rho(0) = \frac{\gamma(0)}{\gamma(0)} = 1$;
2. since $\gamma(k) = \gamma(-k) \rightarrow \rho(k) = \rho(-k)$;
3. then $-1 \leq \rho(k) \leq 1$.

Function $\rho(k)$ measure, therefore, the linear correlation between $X(t)$ and $X(t-k)$.

In a **purely random process** (the variable components of the stochastic process are mutually uncorrelated and identically distributed), the autocovariance $\gamma(k) = 0 \forall k \neq 0$; so we have

$$\rho(k) = 1 \text{ for } k=0$$

and

$$\rho(k) = 0 \text{ for } k \neq 0$$

The examination of this function is fundamental in the study of time series; its study allows to adapt a theoretical model explaining the evolution over time of the phenomenon under investigation.